RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2017

FIRST YEAR [BATCH 2016-19]

Date : 18/05/2017 **PHYSICS (Honours)**

Time : 11 am – 3 pm **Paper : II** Full Marks : 100

[Use a separate Answer Book for each group]

Answer <u>any two</u> questions from each of the Groups A, B and C and <u>any four</u> questions from Group D

Group – A [20 marks]

1. Let $f(x)$ be a function of period 2π such that $f(x)=\dfrac{x}{2}$ over the interval $0<x<2\pi$.
   a) Sketch a graph of $f(x)$ in the interval $0<x<4\pi$. 2
   b) Show that the Fourier series for $f(x)$ in the interval $0<x<2\pi$ is $\dfrac{\pi}{2}-\left[\sin x+\dfrac{1}{2}\sin 2x+\dfrac{1}{3}\sin 3x+\cdots\right]$. 4
   c) By giving an appropriate value of *x* show that $\dfrac{\pi}{4}=1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\dfrac{1}{9}-\cdots$. 2
   d) Write Dirichlet conditions for Fourier series. 2

2. a) Consider the Maxwell's equation: $\vec{\nabla}\cdot\vec{E}(\vec{r},t)=\dfrac{1}{\varepsilon_0}f(\vec{r},t)$. Let $\vec{\varepsilon}(\vec{k},t)$, $f(\vec{k},t)$ are the fourier spatial transform of $\vec{E}(\vec{r},t)$, $f(\vec{r},t)$ respectively. Write the equation in reciprocal or Fourier space. 2
   b) Find the Fourier transform of e^{-ax^2} and comment on result. Graphically draw both the functions (given and result) and show if any change. 3+1+1
   c) Using properties of Dirac function show that $f(x)\,\delta(x-a)=f(a)\,\delta(x-a)$. 1½
   d) Evaluate $\displaystyle\int_{-\infty}^{\infty}e^{-5t}\delta(-z)\,dt$. 1½

3. Consider the Laplace's equation $u_{xx}+u_{yy}=0$. $u(x,y)$ is the steady state temperature at any point in the region $x>0,\ 0<y<h$.
   a) Assume a solution of the form $u(x,y)=F(x)G(y)$, and obtain two ordinary differential equations taking the separation constant to be σ.
   Use the boundary conditions: $u(x,0)=0,\ u(x,h)=0,\ x>0$; to show that $G(0)=0$ and $G(h)=0$. 1+1
   b) Show that $\sigma=0$ and $\sigma<0$ leads to trivial solutions. 2
   c) Show that for $\sigma>0$ the nontrivial solutions can take the form $G_n(y)=B_n\sin\dfrac{n\pi}{h}y,\ n=1,2,\ldots$ 3
   d) Solve $F''-\sigma F=0$ and show that a sequence of solutions $\{F_n(x)\}$ are obtained. 1
   e) Consider $u_n=F_nG_n$. Use the limiting condition $u(x,y)\to 0$ as $x\to\infty,\ 0<y<h$ to find u_n. 2

4. a) Solve by the method of separation of variables: $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$; given that $u(x,0)=4e^{-x}$. 4

b) Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ under the following conditions: $u(0,t)=0;\ u(l,t)=0$; $u(x,0)=a\sin\left(\frac{\pi x}{l}\right);\ \frac{\partial u}{\partial t}(x,0)=0$. 6

Group – B [20 Marks]

5. a) A particle of mass m moves in a central force-field, $\vec{F}=-\hat{r}\frac{k}{r^2}$, ($k > 0$, constant)

i) Show that the total energy E is a constant of motion.

ii) Write down equations of motion in plane polar coordinates (r,θ) and show that they can be reduced to a one-dimensional equation of motion in an effective filed of force given by, $F_{eff}(r)=\frac{-k}{r^2}+\frac{mh^2}{r^3}$, where $h=r^2\dot{\theta}$. Hence obtain the effective potential $u_{eff}(r)$. 2+2

b) i) Sketch the graph of $u_{eff}(r)$ vs r, and use it to discuss the nature of the orbits for $E>0,\ E=0$ and $E<0$, with suitable diagrams.

ii) Calculate the positions of the turning points for each of the above cases.

iii) Hence, calculate the total energy of the particle moving in a circular orbit. 3+2+1

6. a) A Cartesian coordinate system R with unit vectors $(\hat{e}_1,\hat{e}_2,\hat{e}_3)$ rotates with respect to a stationary coordinate system S having common origin, with an angular velocity $\vec{w}$. The position vector of a moving particle is $\vec{r}$ in both S and R.

i) Show that, if $\vec{u}=\left(\frac{d\vec{r}}{dt}\right)_R$ and $\vec{v}=\left(\frac{d\vec{r}}{dt}\right)_S$, are the velocities of the particle is R and S respectively, $\vec{v}=\vec{u}+\vec{w}\times\vec{r}$. [You may assume the result: $\left(\frac{d\hat{e}_i}{dt}\right)_S=\vec{w}\times\hat{e}_i,\ i=1,2,3$].

ii) Hence obtain an expression for the acceleration of the particle as observed from S, and interpret clearly the various terms appearing therein. 2+3

b) Use the results of 2(a) to write down the equation of motion of the particle in the rotating frame R, if $\vec{F}$ is the force on the particle. Explain the nature of the various terms in the expression. 3

c) A bug crawls radially outwards from the centre of a disc rotating with an uniform angular velocity w about a perpendicular axis through the centre. If it is to move with a constant speed v, what forces are necessary to keep it doing so? 2

7. a) The equation of motion of a projectile of mass m and instantaneous velocity $\vec{v}$, moving near the earth's surface is, $m\frac{d\vec{v}}{dt}=m\vec{g}-2m\vec{w}\times\vec{v}$, where $\vec{w}$ is the uniform angular velocity of the earth about the N–S axis.

i) Set up a suitable rotating coordinate system at any point P on the surface at latitude λ.

ii) Solve the above equation in this coordinate system to find the position vector $\vec{r}$ as a function of time, with the initial conditions: at $t = 0$, $r=r_0,\ \vec{v}=\vec{u}_0$ (Neglect all w^2 – terms). 2+3

b) A particle moves under the influence of a force field given by $\vec{F} = a\,(\hat{x}\sin wt + \hat{y}\cos wt)$. If the particle be initially at rest, prove that at an instant *t*; work done on the particle is given by $\frac{a^2}{mw^2}(1-\cos wt)$. (Do not ignore the initial conditions). 5

8. Consider a rotating rigid body. Axis of rotation $(\hat{n})$ passes through the origin of the body reference frame *OXYZ*. Let $\vec{r}_i$ be the position vector of *i*-th particle in the body.

a) i) Draw a figure showing *OXYZ*, $\hat{n}, \vec{r}_i$ and write $\vec{r}_i$ in $\hat{i}, \hat{j}, \hat{k}$ basis.

ii) Write $\hat{n}$ in terms of direction cosines.

iii) Deduce the explicit expression for moment of inertia $I_{\hat{n}}$ about axis $\hat{n}$. 2+1+3

b) Moment of inertia of a cube about an axis that passes through the center of mass and center of one face is I_0. Find the moment of inertia about an axis through the center of mass and one corner of the cube. (Use the explicit expression you derived in a(iii)). 4

<u>Group – C</u> [20 Marks]

9. a) State and prove Gauss's theorem of gravitation. 4

b) A particle moves in a circular orbit under the action of a force $f(r) = -\frac{K}{r^2}$. If *K* is suddenly reduced to half its original value, show that the particle would move along a parabola. 3

c) Derive an expression for gravitational field inside a sphere of radius *R*, when the mass density at a point is $\rho = a + br^2$, where *a* & *b* are constants and *r* is the distance of the point from the centre of the sphere. 3

10. a) Differentiate between angle of twist and angle of shear. 1

b) Derive an expression for the couple required to bend a uniform straight metallic strip into an arc of a circle of small radius. 4

c) A cylinder of radius *r* and length *l* is recast into a pipe of same length. If the pipe possesses torsional rigidity which is 19 times greater than that of the original cylinder, calculate the inner radius of the pipe. 2

d) Find the amount of work done in twisting a steel wire of radius 1 mm and length 25 cm through an angle of 45°. Modulus of rigidity of steel being 8×10^{11} C.G.S. unit. 3

11. a) Define surface tension. Find the excess pressure acting on the curved surface of curved membrane. 1+3

b) Explain why water is raised and mercury is depressed in a capillary tube. 2

c) Two soap bubbles of radii r_1 and r_2 coalesce to form a single bubble of radius *R*. If the external pressure is *P*, prove that the surface tension *T* of the soap solution is given by $T = \frac{P(R^3 - r_1^3 - r_2^3)}{4(r_1^2 + r_2^2 - R^2)}$. 4

12. a) Derive coefficient of viscosity. 1

b) What are the different forces one need to consider while studying fluid dynamics? By considering all the forces derive the Euler's equation for fluid dynamics. 6

c) Water is flowing through a capillary tube 40 cm long and of 1 mm internal radius under a constant pressure head of 15 cm of water. Calculate the maximum velocity of water in the tube and verify that the flow is stream lined. Given: for water viscosity = 0.0098 poise. Reynold's number = 1000 and g = 980 cm/sec^2. 3

Group – D [40 Marks]

13. a) What are the characteristic features of stationary waves? Distinguish between stationary and travelling waves. 2

b) State and explain the superposition principle and linearity. 2

c) For a wave in medium, the angular frequency ω and the wave vector $\vec{k}$ are related by:

$$\omega^2 = c^2 k^2 (1 + \alpha k^2).$$

where c and α are constants. Prove that the product of group velocity and phase velocity is given by:

$$v_g \cdot v_p = c^2 (1 + 2\alpha k^2).$$ 3

d) A particle is subjected to two SHMs represented by the equations $x = a_1 \sin \omega t,\ y = a_2 \sin(2\omega t + \delta)$ in a plane acting at right angles to each other. Discuss the formation of Lissajons' figures due to superposition of these two vibrations. 3

14. a) Obtain an expression for velocity of longitudinal waves propagating through a medium of density ρ and bulk modulus of elasticity χ. 4

b) The intensity of sound in a normal conversation is about $3\times10^{-6} W/m^2$, and the frequency of normal human voice is about 1 KHz. Find the amplitude of waves, assuming that the air is at standard conditions (density $\approx 1\ kg/m^3$, speed of sound in air $\approx 340\ m/s$). 2

c) Transverse waves are generated in two uniform steel wires A and B of diameters $10^{-3} m$ and $0.5\times10^{-3} m$, respectively; by attaching their free end to a vibrating source of frequency 500 Hz. Find the ratio of wavelengths if they are stretched with the same tension. 2

d) Using adiabatic gas laws show that the velocity of sound waves through gases is given by $v = \sqrt{\frac{\gamma RT}{M}}$, where $\gamma = \frac{C_P}{C_V}$, ratio of principal heat capacities. 2

15. a) The general expression for transverse displacement $\eta(x,t)$ of a plucked string (clamped at $x = 0$ & $x = L$) is given by:

$$\eta(x,t) = \sum_n c_n \sin\frac{n\pi x}{L} \cos\frac{n\pi vt}{L},$$

where v is the speed of transverse waves on the string.

If the string is plucked at $x = a$, through a height h, so that the initial displacement $\eta(x,0)$ is given by:

$$\eta(x,0) = \frac{h}{a}x, \text{ for } 0 < x < a$$
$$= \frac{h}{L-a}(L-x), \text{ for } a < x < L,$$

then find the coefficients c_n.

Hence prove that the energy of transverse vibration of the string, plucked at $x = \frac{L}{3}$, is given by:

$$E_{total} = \frac{\mu}{L}\left(\frac{9hv}{2\pi}\right)^2 \sum_n \frac{1}{n^2} \sin^2 \frac{n\pi}{3},$$

where μ is the mass per unit length of the string. 4+3

b) Suppose a source, emitting waves of frequency ν and an observer move in the same direction with velocities u and v, respectively. Show that the frequency registered by the observer is given by: 3

$$\nu' = \frac{\nu(c-v)}{c-u} \quad (c \text{ being the wave velocity}).$$

16. a) What are temporal and spatial coherence? 2

b) Obtain the ratio of coherence lengths of two laser sources having frequency widths 2×10^2 Hz and 3×10^2 Hz. 2

c) Explain the formation of coherent sources in the case of Fresnel biprism using a neat diagram. 3

d) In Newton's rings experiment the diameter of 10^{th} ring changes from 1.5 cm to 1.4 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid. 3

17. a) Derive Airy's formula for the intensity distribution of the transmitted beam in case of Fabry-Perot interferometer. Explain the sharpness of fringes obtained with the interferometer in terms of fringe half width. 6

b) Fringes of equal inclination are observed in a Michelson interferometer. As one of the mirrors is moved back 1 mm, 3663 fringes move out from the centre of the pattern. Calculate the wavelength of the light. 2

c) What is the smallest resolvable wavelength difference for a Fabry-Perot interferometer, if the reflectivity of the plates is 95%? (Assume the central wave length to be 5000 $\mathring{A}$.) 2

18. a) Find the intensity expression for the Fraunhofer diffraction pattern of the double slit. Deduce the conditions for maxima and minima. 4+2

b) Find the missing orders in the double slit diffraction pattern if the width of each slit is half of the separation of two consecutive slits. 2

c) A plane transmission grating having 3000 lines/cm gives an angle of diffraction of a line by 30^o in third order. Find the wavelength. 2

19. a) What is meant by the resolving power of optical instruments? Explain Rayleigh criterion for resolution. 3

b) What are half period zones? Obtain the expression for area of the n^{th} zone. 3

c) Obtain the expression for resolving power of a plane diffraction grating. Hence find the width of a grating of 2000 lines/cm to resolve the sodium D_1 and D_2 lines in second order. 4

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